Mathematical Modeling of Environmental Crime Using Level Set Methods

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Problem

We consider an ecological crime model wherein extractors (e.g. loggers) enter a protected national park area to illegally extract resources, while defenders (e.g. law enforcement agencies) patrol and attempt to apprehend them. A previous model specified a circular forest with radially symmetric features [1]. Here, we use the level set method [2] to generalize the setting to protected areas with arbitrary shape and incorporate geographic information. Our aim is to assess the effectiveness of different patrol strategies.

Definitions

- $\psi = \psi(x, y)$ is the **patrol density** function and is chosen by the defender. ψ is limited by a prescribed budget.
- $B(x, y)$ is the known **benefit** the extractor acquires by extracting at (x, y).
- $C(x, y)$ is the **expected cost** the extractor incurs by extracting at (x, y).
- The extractor's **expected profit** at (x, y) is $P(x, y) = B(x, y) - C(x, y)$.
- The **pristine region** is the part of the protected area through which extractors do not travel.

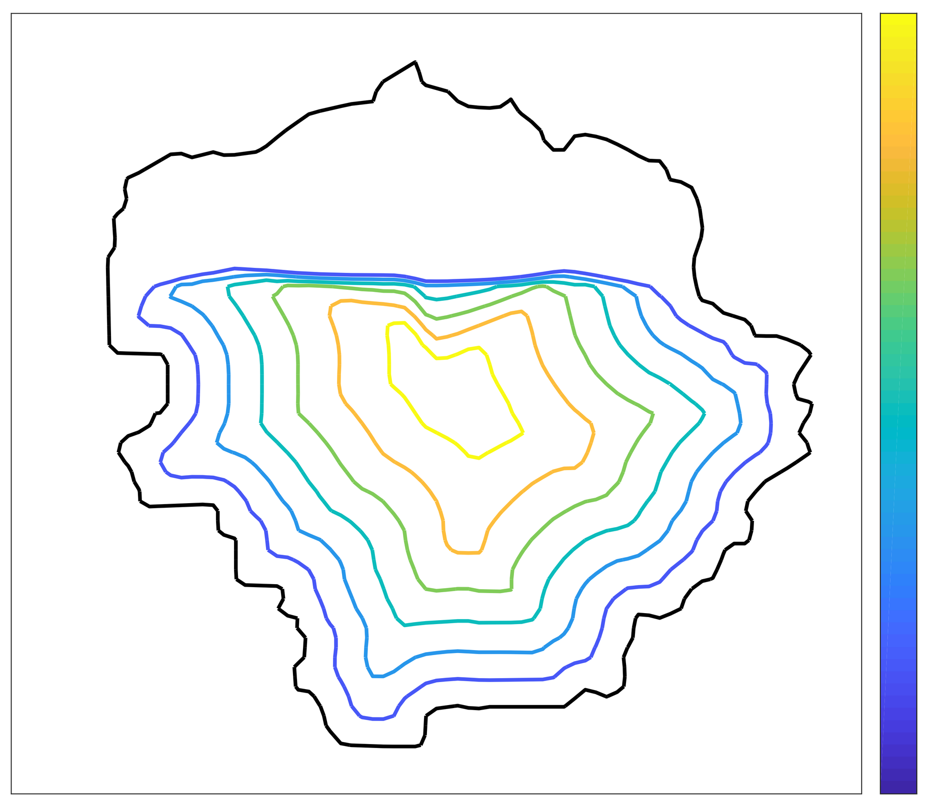


Figure: Expected profit contours for Yosemite National Park, with a mountainous region at the top which increases the cost of movement, and hence lowers the profit.

Expected Cost Function

The **expected cost function** $C(x_0, y_0)$ of extracting at a given point is calculated implicitly from $\phi(x_0, y_0, C) = 0$, where ϕ is an auxiliary function evolved according to

$$\frac{\partial \phi}{\partial C} = -\frac{1}{1/v + \alpha \psi B(x_0, y_0)} |\nabla \phi|.$$

The zero level set of ϕ moves with normal velocity given by the coefficient of $|\nabla \phi|$, the denominator of which has two parts.

1. The first term is based on the **traveling speed**, $v = v(x, y)$, which depends on geographic features.
2. The second term is based on the risk of being caught, and is proportional to the patrol density $\psi = \psi(x, y)$ and to the amount of benefit that would be lost if captured, $B(x_0, y_0)$. α is a factor that allows the travel time and risk-of-capture terms to be compared and weighted against each other.

To find the expected cost everywhere in the protected region, solve the level set equation for each value of benefit B^* to give the cost associated with extracting at the points (x, y) where $B(x, y) = B^*$.

For details on the numerical method we use, see "Numerical Methods for the Level Set Equation with Obstacles: An Application to Problems in Ecological Crime Modeling".

Finding the Non-Pristine Region

1. Once the expected cost function is known, the expected profit $P(x, y)$ can be calculated. We assume that extractors will choose to extract only where the expected profit is within some tolerance of the maximum possible profit.
2. We assume that extractors will take a path that is within some tolerance of the minimum cost path from the high-profit region to the boundary. Only some segments of the boundary will be accessible from the high-profit region within this cost constraint.

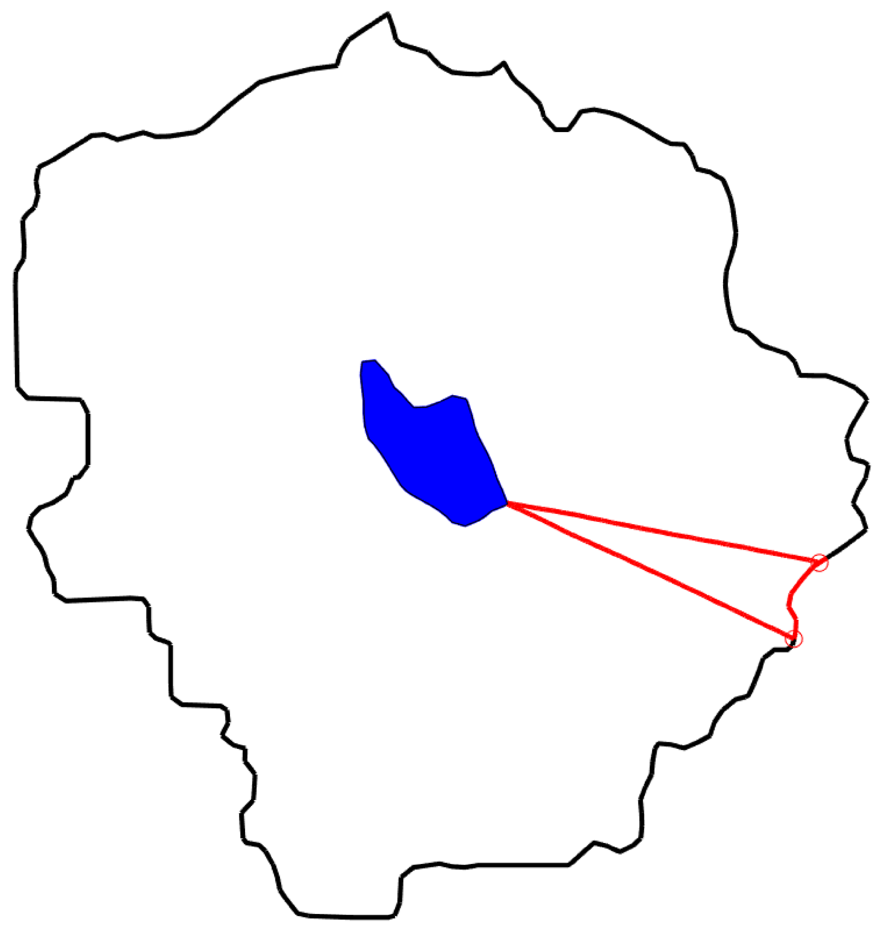


Figure: Yosemite National Park with high profit region in blue (step 1) and paths to one segment of the boundary (steps 2,3)

3. We search for the paths from the high-profit region to the boundary that incur the maximum acceptable cost. These paths bound the range of possible exit routes.
4. The high-profit region, and the bounding paths from it to the park boundary, together bound the non-pristine region, and its area can be calculated.

This algorithm produced the following non-pristine region for Yosemite National Park with uniform patrol.

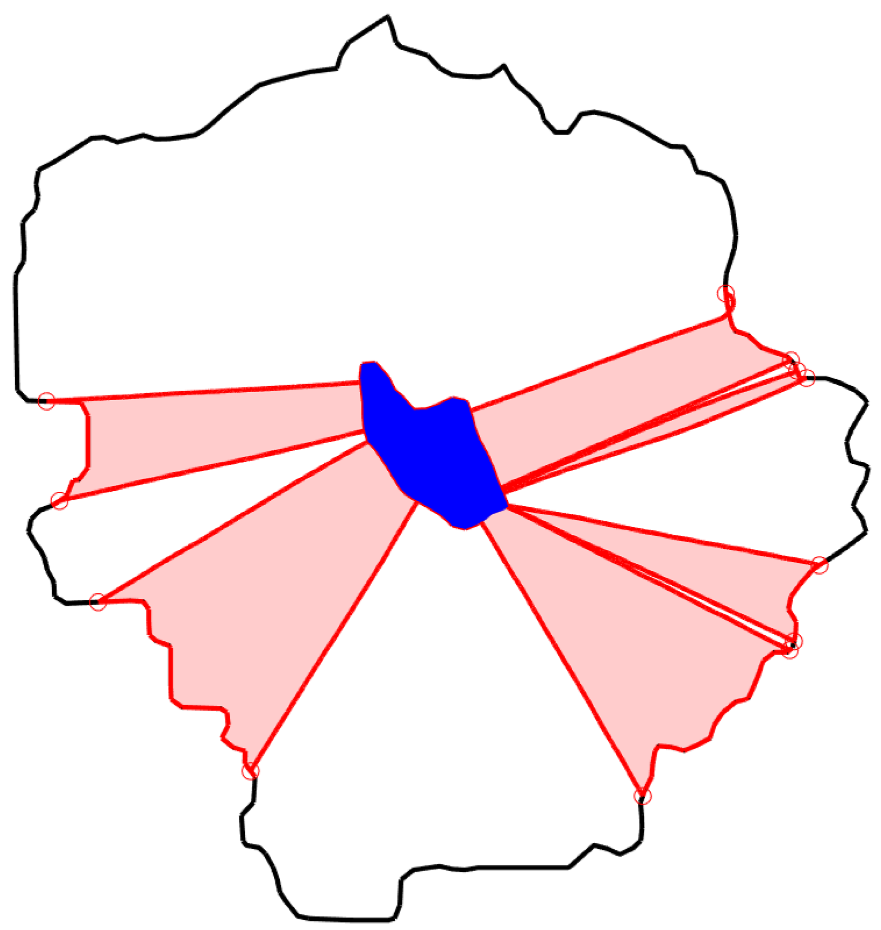
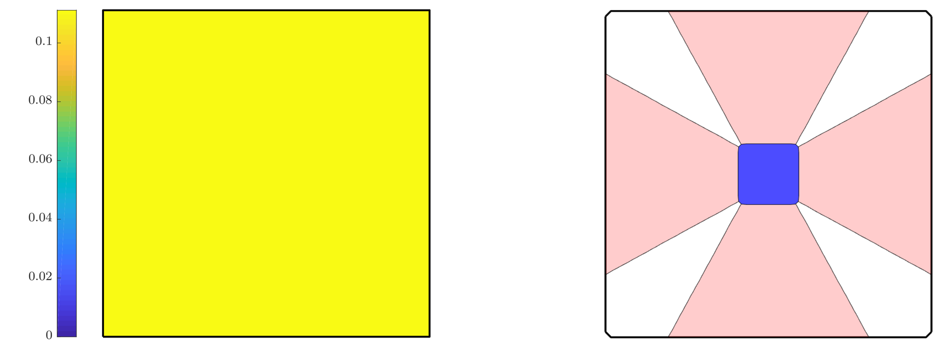


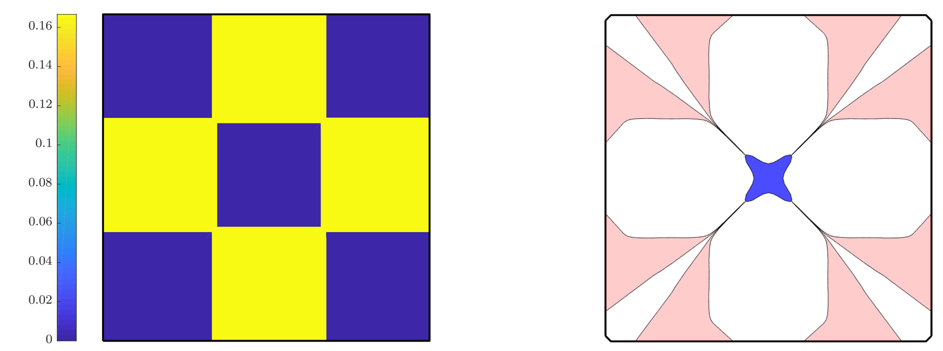
Figure: The high profit region is blue and the non-pristine area is red.

Different Patrols

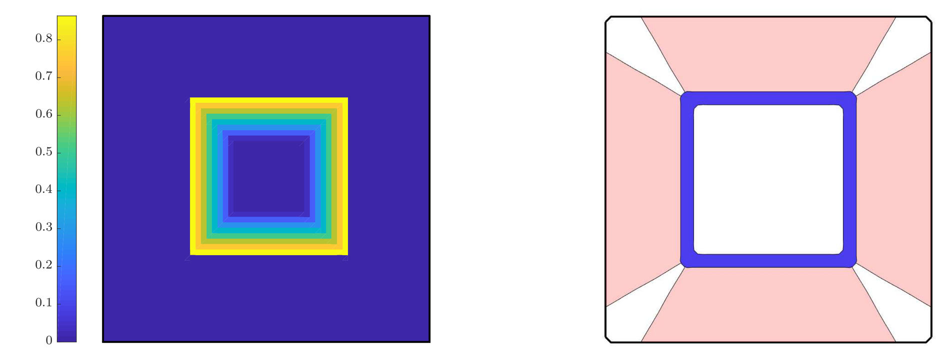
We test our model using three patrol strategies to observe the impact of different strategies on the pristine area, with the same effective coverage. The traveling speed is constant and benefit is proportional to distance from the boundary.



(a) Uniform Patrol



(b) Checker Board Patrol



(c) Band Patrol as in [1]

Figure: The left column shows patrol density ψ (yellow for high density, blue for low density, note different scales). The right column shows the pristine region (white), high-profit region (blue), and exit path region (red).

References

[1] M. Johnson, F. Fang and M. Tambe, *Patrol Strategies to Maximize Pristine Forest Area*, Proceedings of the Association for the Advancement of Artificial Intelligence (2012).

[2] S. Osher, and J. A. Sethian, *Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations*, Journal of Computational Physics, 79, (1988).

Acknowledgements

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